3/PHY–200 (Th) Syllabus–2023

2025

(Nov–Dec)

FYUP : 3rd Semester Examination

MAJOR

PHYSICS

(Mathematical Physics—II and Experimental Physics—III)

PHY–200

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer *any* **eight** questions

1. Obtain the expression for the divergence and curl of a vector field in cylindrical coordinate. 3½+3½=7

2. *(a)* Find the divergence and curl of the following vector : 1+2=3

$$\vec{A} = r\cos\phi\,\hat{e}_r - r\sin\phi\,\hat{e}_\phi$$

(b) State Cauchy's theorem. Verify the theorem using the function $f(z) = z^2$ with the contour C defined as $|z| = 1$. 1+3=4

3. (a) What is the main difference between Taylor series and Laurent series in complex analysis? Find the Laurent series of the function $f(z) = \frac{1}{z(1-z)}$ at $z = 1$. 1+3=4

(b) What are analytic functions? Give an example. What are the conditions for the function $f(z)$ to be analytic in a domain D. 1+2=3

4. (a) Find the residue of the function $f(z) = \frac{z}{z^2+1}$ at $z = i$. 2

(b) State and prove Cauchy's residue theorem. 5

5. (a) Find the singular points of the following differential equation and identify whether they are regular or irregular singular points : 2×2=4

(i) $(x^3 - x)\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 7xy = 0$

(ii) $(1 - x^2)\frac{d^2y}{dx^2} - (x-1)\frac{dy}{dx} + 2y = 0$

(b) Explain Frobenius method of solving differential equation. 3

6. (a) Which types of second-order differential equations can be solved by power series method? 2

(b) Obtain the orthogonality condition of the Legendre's polynomial $P_n(x)$. 5

7. Write down the Hermite differential equation and find its solution using the power series method. 7

8. (a) Deduce the Rodrigue's formula

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$$ 5

(b) Show that $P_n(-x) = (-1)^n P_n(x)$. 2

9. (a) Obtain the following recurrence relation : 3

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

(b) What are symmetric and skew symmetric matrices? Give an example of each. Show that the inverse of a symmetric matrix is also a symmetric matrix. 2+2=4

10. *(a)* If $x = u(1-v)$ and $y = uv$, find the Jacobian $J = \frac{\partial(u, v)}{\partial(x, y)}$ and $J' = \frac{\partial(x, y)}{\partial(u, v)}$ and show that $JJ' = 1$. 2+2+1=5

(b) Write down the Jacobian of implicit function. 2

11. Evaluate the value of $\Gamma(\frac{1}{2})$ and show that $\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$. 5+2=7

12. *(a)* Evaluate the integral

$$\int_0^\infty x^{\frac{1}{4}} e^{-x^{\frac{1}{2}}} dx$$

using gamma function. 4

(b) Evaluate the integral

$$I = \int_0^1 x^5 (1 - x^2) dx$$

using beta and gamma functions. 3

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